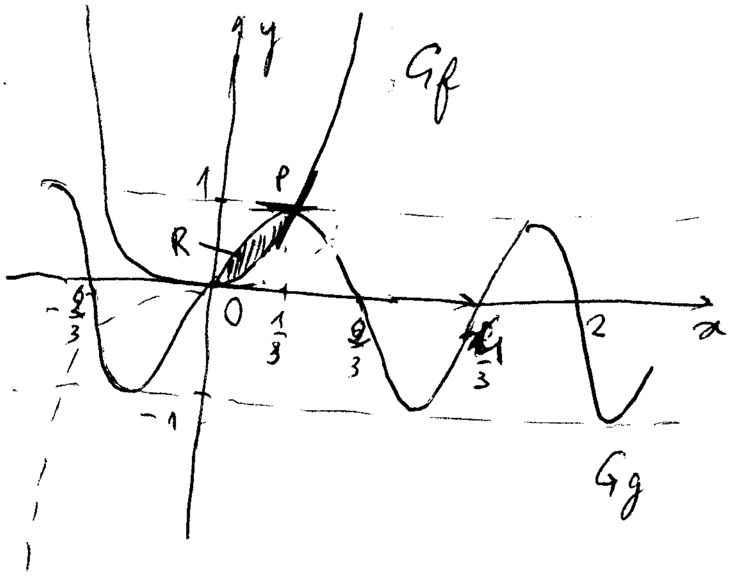
PB. 1

$f(x) = |27x^3|$

$g(x) = \operatorname{sen}\left(\frac{3}{2}\pi x\right)$

1) $\operatorname{sen}\left(\frac{3}{2}\pi(x) + 2\pi k\right) = \operatorname{sen}\left(\frac{3}{2}\pi\left(x + \frac{2\pi}{\frac{3}{2}\pi}\right)\right) = \operatorname{sen}\left[\frac{3}{2}\pi\left(x + \frac{4}{3}\right)\right]$ $\boxed{T = \frac{4}{3}}$



2) $x = \frac{1}{3}$

$f\left(\frac{1}{3}\right) = \left|27 \cdot \frac{1}{27}\right| = 1$

$g\left(\frac{1}{3}\right) = \operatorname{sen}\left(\frac{3}{2}\pi \cdot \frac{1}{3}\right) = \operatorname{sen}\frac{\pi}{2} = 1$

$f'\left(\frac{1}{3}\right) = 81x^2 = 81 \cdot \frac{1}{9} = 9$

$g'\left(\frac{1}{3}\right) = \frac{3}{2}\cos\frac{3}{2}\pi = \frac{3}{2}\cos\left(\frac{3}{2}\pi \cdot \frac{1}{3}\right) = 0$

LE CURVE SI INTERSECANO IN $P\left(\frac{1}{3}; 1\right)$

$f(x) = f'(x_0) \cdot (x - x_0) + f(x_0) \Rightarrow$

tg a G_x: $y = 1 + \left(x - \frac{1}{3}\right) \cdot 9 = 1 + 9x - 3 = \boxed{9x - 2}$

tg. a G_y: $y = 1 + \left(x - \frac{1}{3}\right) \cdot 0 \Rightarrow \boxed{y = 1}$

$\operatorname{tg}\alpha = \left|\frac{m - m'}{1 + mm'}\right| = \frac{9 - 0}{1 + 9 \cdot 0} = 9.$

$\alpha = \operatorname{arc}\operatorname{tg} 9 = \boxed{83° \, 39'}$

3) $\mathcal{A} = \int_0^{\frac{1}{3}} \left[\operatorname{sen}\left(\frac{3}{2}\pi x\right) - 27x^3\right] dx = \left[\frac{-\cos\left(\frac{3}{2}\pi x\right)}{\frac{3}{2}\pi} - 27\frac{x^4}{4}\right]_0^{\frac{1}{3}} =$

SIAMO IN $x > 0$

$= \frac{-\cos\left(\frac{3}{2}\pi \cdot \frac{1}{3}\right) + \cos\left(\frac{3}{2}\pi \cdot 0\right)}{\frac{3}{2}\pi} - \frac{27}{4}\left(\frac{1}{3}\right)^4 = \frac{-0 + 1}{\frac{3}{2}\pi} - \frac{27}{4} \cdot \frac{1}{81} = \frac{2}{3\pi} - \frac{1}{12} =$

$= \boxed{\frac{8 - \pi}{12\pi}}$

5) $V_S = \pi\int_0^{1/3}\left[\sin^2\left(\frac{3}{2}\pi x\right) - \left(27x^3\right)^2\right]dx$

$V_T = \pi\int_0^1 \left\{\frac{1}{9}\sqrt[3]{y^2} - \left[\frac{2}{3\pi}\arcsin y\right]^2\right\}dy$

$y = 27x^3$

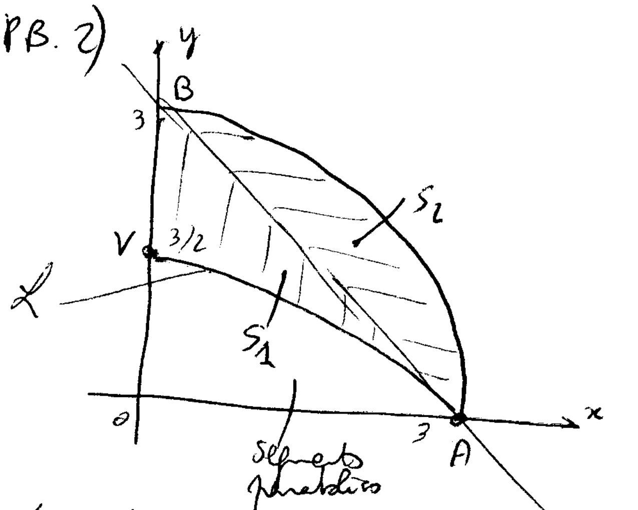
$x = \sqrt[3]{\frac{y}{27}} = \frac{1}{3}\sqrt[3]{y}$

$y = \sin\left(\frac{3}{2}\pi x\right)$

$\frac{3}{2}\pi x = \arcsin y$

$x = \frac{2}{3\pi}\arcsin y$

PB. 2)



$A(3;0)$

$B(0;3)$

$P: x^2 = 9-6y$

$6y = 9 - x^2$

$y = -\frac{1}{6}x^2 + \frac{3}{2}$ $\quad V\left(0;+\frac{3}{2}\right)$

per $A(3;0)$

1) $y' = -\frac{1}{6}\cdot 2x = -\frac{1}{3}x$

$f(3) = -\frac{1}{6}\cdot 9 + \frac{3}{2} = -\frac{3}{2} + \frac{3}{2} = 0$

$f'(3) = -\frac{1}{3}\cdot 3 = -1$

$y - 0 = -1(x-3)$

$\begin{cases} \boxed{y = -x+3} \text{ retta tg.} \\ x = 0 \end{cases}$

$\begin{cases} y = 3 \\ x = 0 \end{cases}$ $B(0;3)$ è la retta AB!

Area quarto di cerchio $= \frac{9\pi}{4}$

Area $A\hat{O}B = \frac{9}{2}$

Area $S_2 = \frac{9}{4}\pi - \frac{9}{2} = \boxed{\frac{9(\pi-2)}{4}}$

Area segmento parabolico $= \int_0^3\left(-\frac{1}{6}x^2 + \frac{3}{2}\right)dx =$

$= \left[-\frac{x^3}{18} + \frac{3}{2}x\right]_0^3 =$

$= -\frac{27}{18} + \frac{9}{2} =$

$= \frac{9}{2} - \frac{3}{2} = \frac{6}{2} = 3$

oppure:

Area segmento parabolico $=$

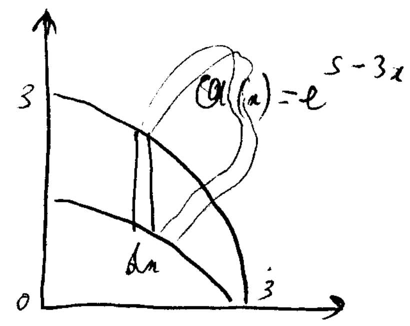
$= \frac{2}{3}\cdot 3\cdot\frac{3}{2} = 3$

Area S_1 = Area $A\hat{O}B$ − Area segmento parabolico $=$

$= \frac{9}{2} - 3 = \boxed{\frac{3}{2}}$

2) METODO DEGLI INDIVISIBILI

$$V=\int_0^3 \left(e^{5-3x}\right)dx = \left[\frac{e^{5-3x}}{-3}\right]_0^3 =$$



$$dV=\left(e^{5-3x}\right)dx$$

$$= \frac{e^{5-9}}{-3} - \frac{e^{5-0}}{-3} = -\frac{e^{-4}}{3}+\frac{e^{5}}{3} =$$

$$=\frac{1}{3}\left(e^5-\frac{1}{e^4}\right) = \boxed{\frac{e^9-1}{3e^4}}$$

3) $V_{rotazione} = V_{SEMISFERA} - V_{PARABOLOIDE} =$

$$= \frac{2}{3}\pi R^3 - \pi\int_0^3\left(-\frac{1}{6}x^2+\frac{3}{2}\right)^2 dx =$$

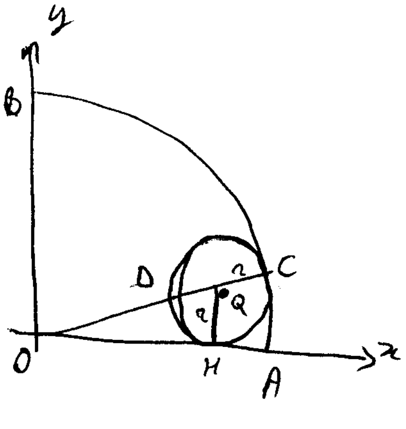
$$= \frac{2}{\cancel{3}}\pi\cdot \cancel{27}^{\,9} - \pi\int_0^3\left[+\frac{x^4}{36}+\frac{9}{4}-\frac{1}{2}x^2\right]dx =$$

$$= 18\pi - \pi\left[\frac{x^5}{180}+\frac{9}{4}x-\frac{x^3}{6}\right]_0^3 =$$

$$= 18\pi - \pi\left[\frac{\cancel{243}^{\,27}}{\cancel{180}_{\,20}}+\frac{27}{4}-\frac{\cancel{27}^{\,9}}{\cancel{6}_{\,2}}\right] =$$

$$= 18\pi - \pi\left(\frac{27+135-90}{20}\right) = 18\pi - \pi\frac{\cancel{72}^{\,18}}{\cancel{20}_{\,5}} = \left(\frac{90-18}{5}\right)\pi = \boxed{\frac{72}{5}\pi}$$

4)



$\overline{QH} = \overline{CQ} = \overline{QD} = r \quad (>0)$

$\overline{OC} = 3$

$\overline{OD} = 3-2r$

$\overline{OQ} = 3-r$

$$\overline{OH}^2 = \overline{OQ}^2 - \overline{QH}^2 = (3-r)^2 - r^2 =$$
$$= 9+\cancel{r^2}-6r-\cancel{r^2} = 9-6r$$

$$Q\left(\sqrt{9-6r};\ r\right)$$

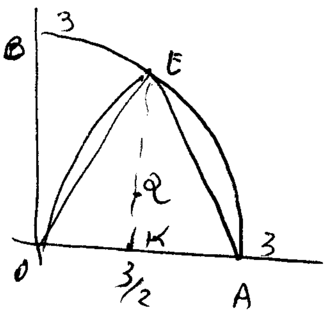
$$\begin{cases} x_Q = \sqrt{9-6r} \\ y_Q = r \end{cases}$$

ELIMINO LA r $\rightarrow$ $x=\sqrt{9-6y}$

$\begin{cases} x^2 = 9 - 6y \\ x > 0 \end{cases}$

$6y = 9 - x^2$

$\boxed{y = -\frac{1}{6}x^2 + \frac{3}{2}}$ Ok!



$\overset{\Delta}{AOE}$ è un triangolo equilatero di raggio $3 = \overline{AO}$

ascissa di $Q = \frac{3}{2}$

ordinata di $Q = \frac{\sqrt{3}}{2}$

$\boxed{Q\left(\frac{3}{2}; \frac{\sqrt{3}}{2}\right)}$

$\overline{EK} = 3\frac{\sqrt{3}}{2}$

Q è BARICENTRO

$\overline{QK} = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

oppure $\overline{QK}$ = raggio circonf. inscritta nel triangolo equilatero

$\overline{QK} = \frac{\mathcal{A}}{p} = \frac{\frac{\cancel{9}\sqrt{3}}{4}}{\frac{\cancel{3}\cdot\cancel{3}}{2}} = \frac{\sqrt{3}}{2}$

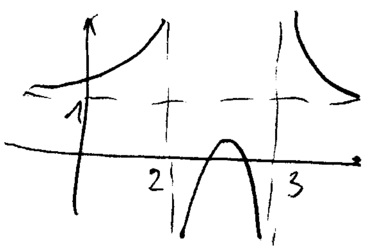
QUESITI

1) è il rapporto incrementale di $y = 5x^4$ in $x_0 = \frac{1}{2}$

$y' = 20x^3$ $\quad y'\left(\frac{1}{2}\right) = 20 \cdot \frac{1}{8} = \boxed{\frac{5}{2}}$

INFATTI, USANDO L'HÔPITAL: $\lim_{h \to 0} \frac{5 \cdot 4 \left(\frac{1}{2} + h\right)^3}{1} = 5 \cdot 4 \cdot \frac{1}{8} = \boxed{\frac{5}{2}}$

2) ASINTOTO È UNA RETTA CHE SI AVVICINA A UNA CURVA SENZA MAI TOCCARLA



es. $y = \frac{x^2}{(x-2)(x-3)} = \frac{x^2}{x^2 - 5x + 6}$

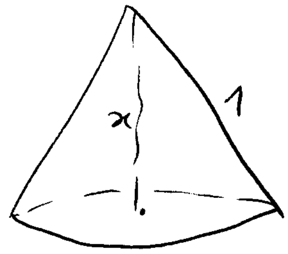
3) $s(t) = 20\left(2e^{-\frac{t}{2}} + t - 2\right)$

$v(t) = s'(t) = 20\left[2 \cdot \left(-\frac{1}{2}\right) e^{-\frac{t}{2}} + 1\right] = 20\left[-e^{-\frac{t}{2}} \quad \right]$

$a(t) = v'(t) = 20\left[\frac{1}{2} e^{-t/2} + 1\right]$

$a(4) = 20\left[\frac{1}{2} e^{-4/2} \quad \right] = \boxed{10/e^2 = (10e^{-2})}$

4) VOLUME DEL CONO MASSIMO DI apotema $= 1$



altezza $= x$ $\qquad 0 < x < 1$

raggio$^2 = 1 - x^2$

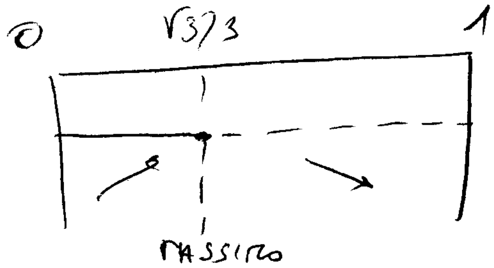
$$V(x) = \frac{1}{3}(1-x^2)\pi x = \frac{(x - x^3)\pi}{3}$$

$$V'(x) = \frac{(1-3x^2)\pi}{3} \geq 0$$

$3x^2 - 1 \leq 0$

$x^2 \leq \frac{1}{3}$

$-\frac{\sqrt{3}}{2} \leq x \leq +\frac{\sqrt{3}}{3}$



MASSIMO

$\boxed{x = \frac{\sqrt{3}}{3}}$

$$V\left(\frac{\sqrt{3}}{3}\right) = \frac{\frac{\sqrt{3}}{3} - \left(\frac{\sqrt{3}}{3}\right)^3}{3}\pi = \frac{\left(\frac{\sqrt{3}}{3} - \frac{\cancel{3}^{1}\sqrt{3}}{\cancel{27}_{9}}\right)\pi}{3} =$$

$$= \frac{\left(\frac{3\sqrt{3} - \sqrt{3}}{9}\right)\pi}{3} = \boxed{\frac{2\sqrt{3}}{27}\pi}\ m^3 = 0{,}1283\pi\ m^3 = \boxed{128{,}3\pi \text{ litri}} =$$

$= \boxed{403{,}9 \text{ litri}}$

5) n punti

I SEGMENTI SONO LE COMBINAZIONI DI n OGGETTI A 2 A 2

$$\binom{n}{2} = \frac{n(n-1)}{2 \cdot 1} = \boxed{\frac{n(n-1)}{2}}$$

(DIAGONALI DI UN POLIGONO DI n LATI PIÙ I LATI)

I TRIANGOLI SONO LE COMBINAZIONI DI n OGGETTI A 3 A 3

$$\binom{n}{3} = \frac{n(n-1)(n-2)}{3 \cdot 2 \cdot 1} = \boxed{\frac{n(n-1)(n-2)}{6}}$$

I TETRAEDRI SONO LE COMBINAZIONI DI n OGGETTI A 4 A 4

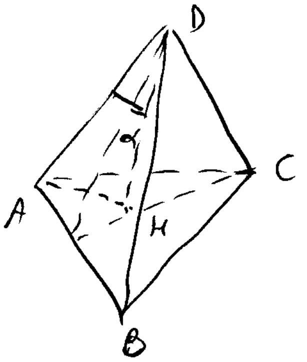
$$\binom{n}{4} = \frac{n(n-1)(n-2)(n-3)}{4 \cdot 3 \cdot 2 \cdot 1} = \boxed{\frac{n(n-1)(n-2)(n-3)}{24}}$$

6) $f(x) = 5\,\text{sen}\,x\cos x + \cos^2 x - \text{sen}^2 x - \frac{5}{2}\,\text{sen}\,2x - \cos 2x - 17 =$

$= 5\,\cancel{\text{sen}\,x\cos x} + \cancel{\cos^2 x} - \cancel{\text{sen}^2 x} - \frac{5}{\cancel{2}}\cdot \cancel{2}\,\text{sen}\,x\,\cancel{\cos x} -$

$- (\cancel{\cos^2 x} - \cancel{\text{sen}^2 x}) - 17 = -17$

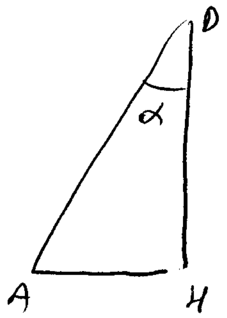
$\boxed{f'(x) = 0}$

7) 

CONSIDERO IL TRIANGOLO $\overset{\Delta}{ADH}$

$\overline{AH}$ è il RAGGIO DELLA CIRCONF. CIRCOSCRITTA AL TRIANGOLO EQUILATERO DI BASE DI LATO l

$A_b = \frac{l^2\sqrt{3}}{4}$



$\overline{AH} = R = \frac{abc}{4A_b} = \frac{l^3}{\cancel{4}l^2\frac{\sqrt{3}}{\cancel{4}}} = \frac{l}{\sqrt{3}} = \frac{1}{3}l\sqrt{3}$

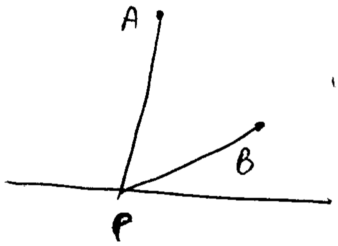
$\overline{DH} = \sqrt{\overline{AD}^2 - \overline{AH}^2} = \sqrt{l^2 - \frac{1}{9}l^2\cdot 3} = \sqrt{l^2 - \frac{l^2}{3}} = \sqrt{\frac{2}{3}l^2} = \frac{\sqrt{6}}{3}l$

$\overline{DH} = \overline{AD}\cos\alpha \to \cos\alpha = \frac{\overline{DH}}{\overline{AD}} = \frac{\sqrt{6}}{3} \cong 0{,}816$

oppure $\overline{AH} = \overline{AD}\,\text{sen}\,\alpha \to \text{sen}\,\alpha = \frac{\overline{AH}}{\overline{AD}} = \frac{\sqrt{3}}{3} \cong 0{,}577$

$\boxed{\alpha = 35°15'52''}$

8) valor medio $= \frac{\int_1^e \frac{1}{x}\,dx}{e-1} = \frac{\left[\ln|x|\right]_1^e}{e-1} = \frac{\ln e - \cancel{\ln 1}}{e-1} = \boxed{\frac{1}{e-1}}$

9) 

SIANO $A(a;b)$ $B(c;d)$ $P(x;0)$ come retta piglio l'asse x

$f(x) = \overline{AP} + \overline{PB} = \sqrt{(a-x)^2 + b^2} + \sqrt{(c-x)^2 + d^2} =$

$= \sqrt{x^2 - 2ax + a^2 + b^2} + \sqrt{x^2 - 2cx + c^2 + d^2}$

7

$$f'(x) = \frac{\cancel{2}x - \cancel{2}a}{\cancel{2}\sqrt{x^2-2ax+a^2+b^2}} + \frac{\cancel{2}x-\cancel{2}c}{\cancel{2}\sqrt{x^2-2cx+c^2+d^2}} =$$

$$= \frac{(x-a)\sqrt{x^2-2cx+c^2+d^2} + (x-c)\sqrt{x^2-2ax+a^2+b^2}}{\sqrt{\cdots}\cdot\sqrt{\cdots}} \geq 0$$

$$(x-a)\sqrt{x^2-2cx+c^2+d^2} \geq (c-x)\sqrt{x^2-2ax+a^2+b^2}$$

$$(x^2-2ax+a^2)(x^2-2cx+c^2+d^2) \geq (c^2-2cx+x^2)(x^2-2ax+a^2+b^2)$$

$$\cancel{x^4}-\cancel{2cx^3}+\cancel{c^2x^2}+d^2x^2-\cancel{2ax^3}+\cancel{4acx^2}-\cancel{2ac^2x}-2ad^2x+$$
$$+\cancel{a^2x^2}-2a^2cx+\cancel{a^2c^2}+a^2d^2 \geq \cancel{c^2x^2}-\cancel{2ac^2x}+\cancel{a^2c^2}+b^2c^2-$$
$$-\cancel{2cx^3}+\cancel{4acx^2}-2ca^2x-2cb^2x+\cancel{x^4}-\cancel{2ax^3}+$$
$$+\cancel{a^2x^2}+b^2x^2$$

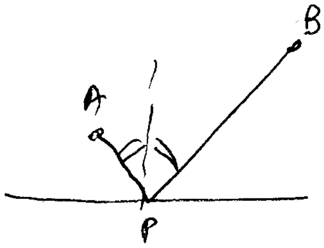
$$(d^2-c^2)x^2+2x(\cancel{ac^2}+a^2c+\cancel{b^2c}-\cancel{ac^2}-ad^2-\cancel{cb^2}) \geq 0$$

$x_1=0$

$$x_2 = \frac{2(a^2c+\cancel{b^2c}-ad^2-\cancel{cb^2})}{d^2-c^2} \Rightarrow x \leq 0 \cup x \geq x_2$$

x_2 È IL MASSIMO CERCATO

$$\boxed{x_2 = 2\left(\frac{a^2c-ad^2}{d^2-c^2}\right)}$$



CORRISPONDE ALLA LEGGE DELLA RIFLESSIONE: ANGOLO DI INCIDENZA PARI ALL'ANGOLO DI RIFLESSIONE!

10) x^2+1 SEMPRE MAGGIORE DI 1

$\ln(x^2+1)$ SEMPRE POSITIVO

$\cos \ln(x^2+1)$ NON SEMPRE POSITIVO (es $\cos\pi$) la D NO

$\operatorname{sen} \ln(x^2+1)$ NON SEMPRE POSITIVO (es $(\operatorname{sen}\frac{3}{2}\pi)$ la C NO

È LA A

D

$$\cos[\ln(x^2+1)] > 0$$

$$-\frac{\pi}{2}+k\pi \leq \ln(x^2+1) \leq \frac{\pi}{2}+k\pi$$

MINORE DI -1 MAGGIORE DI 1

$\forall x$

$$\operatorname{sen}[\cos(x^2+1)] > 0$$

$$0+2k\pi < \cos(x^2+1) < \pi+2k\pi$$

MAGGIORE DI ZERO MAGGIORE DI 1

$$-\frac{\pi}{2}+k\pi \leq x^2+1 < \frac{\pi}{2}+k\pi$$

SEMPRE NON SEMPRE