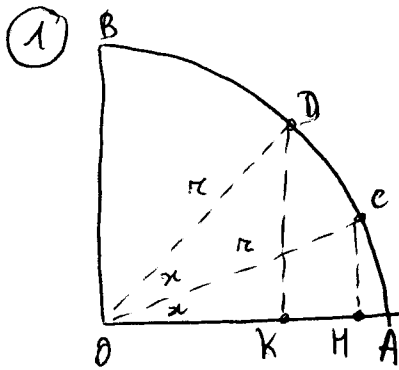


PROBLEMI CON I LIMITI



1) SIA UN QUARTO DI CERCCHIO DI RAGGIO r . SI PRENDANO DUE RAGGI \overline{OC} E \overline{OD} TALI CHE $\widehat{AOC} = \widehat{COD}$. DETERMINARE

$$\lim_{C \rightarrow A} \frac{HK}{AH}$$

pongo $\widehat{AOC} = x$, $0 < x < \pi/4$

$$HK = \overline{OK} - \overline{OH} = r \cos x - r \cos 2x = r(\cos x - \cos 2x)$$

$$AH = r - r \cos x = r(1 - \cos x)$$

$$\lim_{x \rightarrow 0} \frac{r(\cos x - \cos 2x)}{r(1 - \cos x)} = \lim_{x \rightarrow 0} \frac{(\cos x - 2\cos^2 x + 1)}{(1 - \cos x)} =$$

$$= \lim_{x \rightarrow 0} \frac{(2\cos x + 1)(\cos x - 1)}{(\cos x - 1)} = 2 + 1 = \boxed{3}$$

2) SONO DATE LE PARABOLE DI EQ. $y = x^2$ ED $y = \frac{1}{2}x^2$, E SI CONSIDERANO LE RETTE $y = mx$ ED $y = -m$. DETTE A E B, I PUNTI DI INTERSEZIONE TRA LE PARABOLE ED $y = mx$, E DETTE A' E B' LE LORO PROIEZIONI SU $y = -m$, TROVARE:

$$\lim_{m \rightarrow 0} \frac{S_{\triangle CAB'}}{S_{\triangle ABB'}}$$

$$A \begin{cases} y = mx \\ y = x^2 \end{cases}$$

$$B \begin{cases} y = mx \\ y = \frac{1}{2}x^2 \end{cases}$$

$$A(m; m^2)$$

$$B(2m; 2m^2)$$

$$A'(m; -m)$$

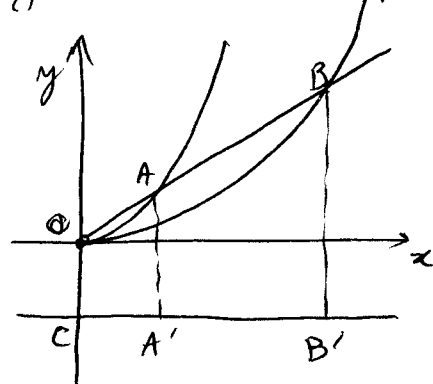
$$B'(2m; -2m)$$

$$\overline{AA'} = m^2 + m$$

$$\overline{CB'} = 2m$$

$$\overline{A'B'} = 2m - m = m$$

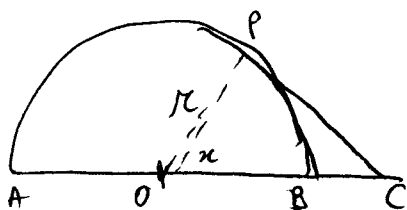
$$\overline{BB'} = 2m^2 + m$$



$$\lim_{m \rightarrow 0} \frac{S_{\triangle CAB'}}{S_{\triangle ABB'}} = \lim_{m \rightarrow 0} \frac{m^2(m+1)}{\frac{1}{2}m^4(2m+1)} = 2 \lim_{m \rightarrow 0} \left(\frac{m+1}{2m+1} \right) = 2 \cdot \frac{1}{2} = \boxed{2}$$

3) È DATA LA SEMICIRCONFERENZA DI DIAMETRO \overline{AB} ED IL SUO RAGGIO \overline{OP} . DETTA T LA TANGENTE IN P CHE INTERSECA \overline{AB} IN C, TROVARE:

$$\lim_{P \rightarrow B} \frac{2 \overline{BC} \cdot \overline{OP}}{\overline{PB}^2}$$



$$\widehat{POC} = x$$

$$\frac{\overline{PC}}{\overline{OC}} = \frac{\overline{OP}}{\overline{OC}} = \frac{r}{\cos x}; \quad \overline{BC} = \frac{r}{\cos x} - r = \frac{r(1 - \cos x)}{\cos x} \quad \overline{PB} = rx$$

$$\lim_{x \rightarrow 0} \frac{2x(1 - \cos x)}{\cos x} \cdot \frac{r}{r^2 x^2} = 2 \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} \frac{1}{\cos x} = \boxed{1}$$

4) DATO IL TRIANGOLO $\triangle ABC$, RETTANGOLO IN A, SI TROVI $\lim_{\widehat{ABC} \rightarrow 0} \frac{\overline{AC}^2}{\overline{BC}(\overline{BC} - \overline{AB})}$

$$\widehat{ABC} = x$$

$$\overline{AB} = \overline{BC} \cos x$$

$$\overline{AC} = \overline{BC} \sin x$$

$$\lim_{x \rightarrow 0} \frac{\overline{BC}^2 \sin^2 x}{\overline{BC}^2 (1 - \cos x)} = \boxed{2}$$

